hep-th/0005093
IP/BBSR/2000-16
IITK-HEP-00-02

# On Charged Strings and their Networks

**Alok Kumar**[1] **and Sudipta Mukherji**[2]

*1. Institute of Physics,*
*Bhubaneswar 751 005, INDIA*
*email: kumar@iopb.res.in*
*2. Physics Department,*
*Indian Institute of Tenchnology,*
*Kanpur 208 016, INDIA*
*email: mukherji@iitk.ac.in*

**ABSTRACT**

We investigate properties of several string networks in $D < 10$ which carry electric currents as well as electrostatic charge densities. We show the electric-current conservations as well as the force-balance condition of the string tensions on 3-string junctions in these networks. We also show the consistency of the above string networks from their world-volume point of view by comparing the world-volume energy-density with the induced worldsheet energy density of the supergravity solution. Finally, we present new charged macroscopic string solutions in type II theories in $D = 8$ and discuss certain issues related to their network construction.

May 2000

String-Network type BPS states have been analyzed in the past few years in string theory (with or without branes), as well as in other quantum field theories [1] - [13]. On the other hand, superconducting strings have been studied in field theories as they are expected to play important role in the evolution of our universe at early time. Construction of such configurations in string theory requires coupling of macroscopic strings to electromagnetic fields. Electromagnetically charged macroscopic strings are known to exist in heterotic string theory for some time [14]. In a recent paper [15], these were extended to the type IIB string theories. Moreover, it was also shown that BPS networks of such objects with 1/4 supersymmetry can also be constructed.

In this paper we analyze various aspects of such electrically charged string networks. We first show that in the above construction, the electric-currents flowing through the strings are conserved on the 3-string junctions. In the absence of complete supergravity solutions for such networks, these are done by examining the current flowing through them far away from any of the 3-string junction. Thus, although a redistribution of the current flow as well those of electrostatic charges are expected to take place in the complete solution near the junction, we observe that the electric-currents emerging far away from these junctions are conserved. We then argue the consistency of charged string from the point of view of world volume theories as well. We discuss this aspect for charged-string networks which are constructed by applying a Lorentz transformation involving time and an internal direction. It has earlier been pointed out that configurations obtained by applying above transformations represent genuinely charged string network. In particular, by comparing the world-volume and supergravity expression for the energy-densities, we show that indeed a fundamental (charged) string ends on a D-string to form a 3-string junction.

Finally, we present a new charged string solution in 8-dimensional type II theories. These solutions are obtained by observing a mapping between the heterotic and type II supergravity actions, after suitable truncations. Since type II classical solutions also map on to the corresponding solutions in the heterotic theory, the BPS nature of type II solutions is guaranteed. However, it is possible that they preserve different set of supersymmetries than the ones obtained by another mapping between the truncated type IIB and heterotic theory, which was used in [15]. To show the difference between the solutions presented in this paper and that of [15], we notice that in the charged macroscopic string solutions of [14] and [15], the charges are acquired by fields which are identified as Kaluza-Klein (KK) gauge fields: $G_{\mu i} \pm B_{\mu i}$, for $i = 1, 2$, representing the internal directions. In the present case however, charges are assigned within an $SL(3)$ multiplet of gauge fields. Since two such multiplets are formed by combinations $(G_{\mu 1}, B^{NS}_{\mu 2}, B^{RR}_{\mu 2})$ and $(G_{\mu 2}, B^{NS}_{\mu 1}, B^{RR}_{\mu 1})$, the solutions presented in this paper are necessarily different from the ones in [15]. Furthermore, we discuss charge and current conservations around these junctions.

We now start with the discussion of the electric current conservations. Let us consider a charged string solution in D-dimensions which is given by a supergravity configuration with non-zero 2-form $B^a_{\mu\nu}$ and 1-form $A^I_\mu$. Only nonzero components

of $B^a_{\mu\nu}(r)$ are $B^a_{0,(D-1)}(r)$ and that of gauge field $A^I_\mu$ are $A^I_0(r)$ and $A^I_{D-1}(r)$. Here $r$ denotes the radial coordinate in $(D-2)$ dimensions transverse to the string and the superscripts $a, I$ on $B_{\mu\nu}$ and $A_\mu$ distinguish between various two-form and one-form fields respectively. Let us first focus on the 2-form fields. The charges associated to the fields are defined by

$$Z^a = \int d\Omega \, {}^*H^a_\Omega. \tag{1}$$

Here $H^a = dB^a$ and $*$ denotes the Hodge dual. The integral above is taken over the $(D-2)$ transverse directions of the string.

Now let us think of a 3-string junction with its three legs coupling to different values of $B^a$'s, denoted as $B^a_1$, $B^a_2$ and $B^a_3$. The corresponding charges are denoted as $Z^a_1$, $Z^a_2$ and $Z^a_3$. Then, using above identifications, the charge conservation: $Z^a_{(1)} + Z^a_{(2)} = Z^a_{(3)}$ implies

$$\int_\infty d\Omega_{(1)} \, {}^*H^a_{\Omega_{(1)}} + \int_\infty d\Omega_{(2)} \, {}^*H^a_{\Omega_{(2)}} = \int_\infty d\Omega_{(3)} \, {}^*H^a_{\Omega_{(3)}}, \tag{2}$$

where we have now introduced subscripts $\Omega_{(i)}$'s for the angular variable in the transverse space. Also, the subscripts of the integrals imply that they are evaluated far away from the junction. In other words, the charge-conservation condition of the above type follows by evaluating the expression (1) along any one of the string of a 3-string junction (far away from it) and then by sliding the large spherical surface through the junction to the other side while deforming it in a manner to surround the remaining two strings.

Same statements are true for the case of 1-form charges as well, when one considers the charges corresponding to non-zero $A^I_{(D-1)}$'s. The field-strengths corresponding to $A^I_{(D-1)}$ are given by $F^I_{(D-1)r}$. The corresponding charges are given as:

$$J^I_0 = \int d\Omega \, {}^*F^I_{0\Omega} \tag{3}$$

where we have now kept the 'time'-index on the charge to differentiate them with other changes defined below. The charge conservation (which finally amounts to the electric-current conservation) then implies the condition:

$$J^I_{0\,(1)} + J^I_{0\,(2)} = J^I_{0\,(3)} \tag{4}$$

As in the case of 2-form charge conservation condition (2), eqn. (4) now follows from the following equation:

$$\int_\infty d\Omega_{(1)} \, {}^*F^I_{0\Omega_{(1)}} + \int_\infty d\Omega_{(2)} \, {}^*F^I_{0\Omega_{(2)}} = \int_\infty d\Omega_{(3)} \, {}^*F^I_{0\Omega_{(3)}}. \tag{5}$$

Finally we discuss the case of gauge charges associated with 1-form components $A^I_0$'s. Now the nonzero field-strengths are: $F^I_{0r}$ and their Hodge-duals are given as: ${}^*F^I_{(D-1)\Omega}$. The corresponding charges are now given as:

$$q^I_{(D-1)} = \int d\Omega \, {}^*F^I_{(D-1)\Omega}. \tag{6}$$

An important difference between two 1-form charges defined in eqns. (3) and (6) is that the later one depends on the direction along which the string lies as this charge is being measured by the value of a field strength at a large orthogonal distance from the string. The corresponding consistency condition of the charged 3-string junction is the force balance condition, depending on their orientations. We now discuss these aspects for examples presented earlier in [15].

We start by analyzing the 3-string junctions of charged macroscopic strings in $D = 9$, discussed in section-(3.1) of [15]. These are parameterized by a single solution-generating parameter $\alpha$. Moreover, its action can be identified in ten-dimensions simply as a Lorentz-transformation involving time-coordinate $x^0$ and and an internal direction: $x^9$. The consistency of the network of such charged-string solutions is already known [15]. Explicit solution for the electrically charged fundamental string ( henceforth $(1,0)$ string ), appearing in the networks, is presented in section-3.1 of [15]. We only write down the 1-form potentials:

$$\hat{A}^1_t = \frac{C\sinh\alpha\ \cosh\alpha}{2(r^5 + C\cosh^2\alpha)}, \quad \hat{A}^1_8 = 0, \tag{7}$$

$$\hat{A}^2_t = 0, \quad \hat{A}^2_8 = \frac{-C\sinh\alpha}{2(r^5 + C)}. \tag{8}$$

As we notice, the above $(1,0)$-string solution is characterized by two gauge fields for this $D = 9$ example. They come from KK reduction of the ten-dimensional metric and antisymmetric tensor fields. Then the $SL(2,Z)$ duality in ten-dimensions generates, one more gauge field identified as the one coming from the KK reduction of the ten dimensional RR sector antisymmetric tensor. Nonzero gauge field components in the final $(p,q)$-string solution can then be written as:

$$\hat{A}^1_t = \frac{C\Delta_q^{1/2}\sinh\alpha\cosh\alpha}{2(r^5 + C\Delta_q^{1/2}\cosh^2\alpha)}, \quad \hat{A}^1_8 = 0, \tag{9}$$

$$\hat{A}^2_t = 0, \quad \hat{A}^2_{i8} = \frac{-C\sinh\alpha}{2(r^5 + C\Delta_q^{1/2})}(M_0^{-1})_{ij}q_j, \ \ i = 1,2, \tag{10}$$

where $i = 1,2$ stand for the $B_{\mu\nu}$'s in NS-NS and R-R sectors of type IIB in ten dimension and $M$ is a $2\times 2$ matrix parameterizing $SL(2)/SO(2)$ moduli. In the above equations, $\Delta_q = q_i(M_0^{-1})_{ij}q_j$. By denoting the currents originating from the metric and the antisymmetric tensors respectively as $J^1$ and $J^2$, the electric current in a $(p,q)$ string [16] is:

$$J^1 = 0, \quad J^2_i = -C\frac{\sinh\alpha}{2}(M_0^{-1})_{ij}q_j, \ \ i = 1,2. \tag{11}$$

We now observe that, apart from an $O(d,d)$ factor $\sinh\alpha/2$, the above electric currents are proportional to the 2-form charges of the strings. As a result, the electric-current conservations hold at the junctions directly due to the conservation of 2-form charges.

We now discuss the electric-current conservation in string networks constructed by starting with the $(1,0)$ charged-macroscopic strings which are T-dual to the ones we mentioned in the last paragraph. An analysis of the supersymmetry condition in this case implies that such solutions are possible in $D = 8$. One then has six gauge fields, two each from the KK components of $G_{\mu\nu}$, $B^{NS}_{\mu\nu}$ and $B^{R}_{\mu\nu}$. We denote the corresponding electric-currents as $J^1_a, J^{2NS}_a, J^{2R}_a$, $a = 1,2$. Then the results of section-(3.2) in [15] imply the following values of the electric-currents for the $(1,0)$ string solution (in a complex notation):

$$J^1_1 + iJ^1_2 = \frac{\Delta_q^{1/2}}{2}\mathrm{sinh}\alpha e^{i\theta}, \quad J^{2i}_1 + iJ^{2i}_2 = 0 \tag{12}$$

where an extra superscript $i$ in $J^2$, in the second equation above, stands for $NS-NS$ and $RR$ components and the parameter $\theta$ is an angular $O(2,2)$ parameter, identified as a spatial rotation among the compactified coordinates $x^8$ and $x^9$. However the network construction requires this parameter to be identical to the one associated with an $SL(2,Z)$ transformation, namely eqn.(16) of [16], which generates a $(p,q)$-string solution from a $(1,0)$ one.

Then for 3-string junctions, nonzero electric-current along a $(p,q)$-string-prong is given by:

$$J^1_1 + iJ^1_2 = \frac{1}{2}\mathrm{sinh}\alpha e^{\phi/2}(p - q\tau) \tag{13}$$

We then once again see the electric-current conservations in the networks of such strings, following directly from the 2-form charge conservations.

Unlike electric-currents, there is no direct way to examine the status of the consistency of (static) electric-charge densities coming from the gauge field components $A^1_t$ in (9), as these are expected to be redistributed in the full supergravity solution near a 3-string junction. One way to settle the issue is to analyze supergravity solutions of such 3-string junctions. In the absence of these solutions, for the moment, we note that the string tension of the charged macroscopic $(p,q)$-strings discussed above in eqns.(7-11) and (12-13) are given by

$$T_{p,q} = \Delta_q^{1/2} C(\mathrm{cosh}\alpha + 1)/2. \tag{14}$$

Since the $O(d,d)$ parameter is only an overall factor, the tension balance continues to hold for both kinds of string-networks mentioned above. This is because the spatial orientations of these strings in a network are identical to the one for the neutral ones.

In view of applications in our later analysis , we now write down the induced world-sheet energy momentum tensors corresponding to general charged F-string solution [15]. They are given in terms of $O(d,d)$ parameters $\alpha, \beta$:

$$\begin{aligned} T_{00} &= C\mathrm{cosh}\alpha\mathrm{cosh}\beta, \\ T_{11} &= C, \\ T_{01} &= \frac{C}{2}(\mathrm{cosh}\alpha - \mathrm{cosh}\beta). \end{aligned} \tag{15}$$

To get the corresponding answer for the examples considered in (7-11) and (12-13), we have to set $\beta = \pm\alpha$. Therefore, for these configurations, the (00) component of world-sheet stress tensor reduces to

$$(T_{00})_{p,q} = \frac{1}{2} C {\Delta_q}^{\frac{1}{2}} (1 + \cosh 2\alpha). \tag{16}$$

This should not, however, be compared with the tension calculated in (14) as $(T_{00})_{p,q}$ receives contributions from string tension as well as from the gauge fields associated with electrically charged strings. However we note that for $\alpha = 0$, the expression (16) matches with the string tension, as expected in the neutral case.

We now discuss the consistency of the charged string solutions from the point of view of the D-string world-sheet theory. In this context, we consider the example of the $D = 9$ charged string networks formed by a Lorentz-boost, parameterized by the parameter $\alpha$ mentioned above. Then, a classical solution representing a 3-string junction of charged macroscopic strings in the world-sheet theory, is given by the application of the Lorentz-boost on the solution of [4] and can be written as in eqns.(17) and (18) below:

$$\begin{aligned} A_0 &= -g x^1 \cosh\alpha, \qquad \Phi \equiv A_9 = -g x^1 \sinh\alpha, \quad x^1 > 0 \\ &= 0 \qquad\qquad\qquad\qquad\qquad\; = 0, \qquad\qquad\quad x^1 < 0, \end{aligned} \tag{17}$$

where $\Phi$ is the scalar coming from the dimensional reduction of the world-volume gauge field from $D = 10$ to $D = 9$. The choice of the Lorentz-transformation parameter '$\alpha$' is fixed through the results in section-(3.1), in particular (3.13) of [15].

To maintain supersymmetry one has to excite one more world-volume field, identified as the coordinate representing the F-string. Following [3], [4], in this case we have

$$\begin{aligned} X^8 &= -g x^1, \quad x^1 > 0, \\ &= 0, \quad x^1 < 0. \end{aligned} \tag{18}$$

This is a 1/2 supersymmetric solution in the world-volume theory. The supersymmetry condition for this solution is obtained from that of the neutral string by the above Lorentz-transformation.

We now evaluate the energy of this configuration to identify it with the expression for $T_{00}$ of the F-string given in (16). In order to proceed, following [3], we first write down the expression of the Hamiltonian associated with the above configuration. After evaluating the expressions, one gets:

$$H = \frac{1}{2}(1 + \cosh 2\alpha) H_0, \tag{19}$$

where $H_0$ is the Hamiltonian associated with the neutral 3-string junction of [4]. The first term inside the bracket in the right hand side corresponds to the contribution of $X^8$ to the classical action whereas the second term is the combined contribution from

$A_0$ and $X^9$. As a result, the energy expression is modified by a factor $(1+\cosh 2\alpha)/2$, which precisely coincides with the energy density of the charged string given in (16), when retricted to $(1,0)$-string. We have therefore given a world-volume argument in favor of the existence of the 3-string junction solutions of charged macroscopic strings by identifying the relevant variables in the two approaches, namely $T_{00}$ in (16) and $H$ in (19). In the limit $\alpha = 0$, one also reproduces: $H_0 = T_f x^8$, a result following from the analysis of [3, 4], with $T_f$ being the F-string tension. In other words, in [3, 4], the world-volume energy was associated with the string tension of a 'spike'-configuration interpreted as an F-string. We observe that similar interpretation holds in the case of charged strings as well, provided one takes into account the contribution of the charges in the supergravity solution.

We now give explicit construction for some new charged string configuration in eight dimensions. We further discuss how various conservation laws are satisfied around the junction for such strings.

Following [17], we start with a truncated version of eight dimensional type IIB action. In Einstein frame the action can be written as

$$\begin{aligned}
S = \int d^8x[ \quad & R - \frac{1}{2}\{(\partial\sigma)^2 + (\partial\phi_1)^2 + (\partial\phi_2)^2\} \\
& -\frac{1}{12}\{e^{-\phi_1+\frac{1}{\sqrt{3}}\phi_2} {H_3^{(1)}}^2 + e^{\phi_1+\frac{1}{\sqrt{3}}\phi_2} {H_3^{(2)}}^2 + e^{-\frac{2}{\sqrt{3}}\phi_2} {H_3^{(3)}}^2\} \\
& -\frac{1}{4}e^{\sigma}\{e^{-\phi_1-\frac{1}{\sqrt{3}}\phi_2} {F_2^{(1)}}^2 + e^{-\phi_1-\frac{1}{\sqrt{3}}\phi_2} {F_2^{(2)}}^2 + e^{\frac{2}{\sqrt{3}}\phi_2} {F_2^{(3)}}^2\} \\
& -\frac{1}{4}e^{-\sigma}\{e^{\phi_1-\frac{1}{\sqrt{3}}\phi_2} {\mathcal{F}_2^{(1)}}^2 + e^{-\phi_1-\frac{1}{\sqrt{3}}\phi_2} {\mathcal{F}_2^{(2)}}^2 + e^{\frac{2}{\sqrt{3}}\phi_2} {\mathcal{F}_2^{(3)}}^2\}]. \qquad (20)
\end{aligned}$$

We would like to make few comments about the origin of different fields in this action. The action contains three scalars $\sigma$, $\phi_1$ and $\phi_2$. They are certain linear combinations of ten dimensional dilaton, and the two scalars that originate due to compactification from ten to eight dimensions. Three different three-form field strengths are denoted above as $H_3^{(i)}$. Furthermore, there are six two-form field strengths. Out of them $F_2^{(i)}$ come from reduction of various antisymmetric tensors in ten dimension. The other set $\mathcal{F}_2^{(i)}$ have their KK origin. In order to keep our discussion simple, we have set all the other fields to zero including the zero forms (axions) that appear in the eight dimensional action. Various details of eight dimensional type IIB supergravity action can be found in [17, 18]. As discussed previously, type IIB string in eight dimensions has $SL(3,R)$ symmetry. Defining $H_3 = dB_2, F_2 = dA_1$ and $\mathcal{F} = d\mathcal{A}_1$, it is easy to see that (20) is invariant under

$$\begin{aligned}
& g_{\mu\nu} \to g_{\mu\nu}, \quad \sigma \to \sigma, \\
& \mathcal{M} \to \mathbf{\Lambda}\mathcal{M}\mathbf{\Lambda}^T, \ \mathcal{A}_\mathbf{1} \to \mathbf{\Lambda}\mathcal{A}_\mathbf{1}, \\
& \mathbf{A}_1 \to \mathbf{\Lambda}\mathbf{A}_1, \quad \mathbf{B}_2 \to (\mathbf{\Lambda}^{-1})^T\mathbf{B}_2, \qquad (21)
\end{aligned}$$

where $\mathbf{\Lambda}$ is a global $SL(3,R)$ matrix. $\mathcal{M}$ is a matrix with diagonal entries $(e^{-\phi_1+\frac{1}{\sqrt{3}}\phi_2}$, $e^{\phi_1+\frac{1}{\sqrt{3}}\phi_2}, e^{-\frac{2}{\sqrt{3}}\phi_2})$. In (21), $\mathcal{A}_\mathbf{1}$ is defined as three dimensional column matrix with

entries $\mathcal{A}_1^{(1)}$, $\mathcal{A}_1^{(2)}$ and $\mathcal{A}_1^{(3)}$. We have also defined $\mathbf{A}_1$ and $\mathbf{B}_2$ in a similar manner. In the following, we will be using only $SO(3)$ subgroup $\mathbf{\Lambda}$ of $SL(3,R)$. This can be represented by Euler angles $\theta, \phi$ and $\psi$.

The macroscopic string solution of this theory in Einstein frame can be written down as

$$\begin{aligned} ds^2 = \frac{1}{[1+NG(r)]^{\frac{2}{3}}} \qquad & [-dt^2 + (dx^7)^2] + \frac{q^2 G(r)}{4N[1+NG(r)]^{\frac{5}{3}}}[-dt + dx^7]^2 \\ & + [1+NG(r)]^{\frac{1}{3}}(dr^2 + r^2 d\Omega_5^2), \end{aligned} \tag{22}$$

with

$$\begin{aligned} & \phi_1 = -\frac{1}{2}\log[1+NG(r)], \quad \phi_2 = \frac{1}{2\sqrt{3}}\log[1+NG(r)] \\ & {B_{t7}}^{(1)} = -\frac{NG(r)}{[1+NG(r)]}, \quad \mathcal{A}_t^{(2)} = -\mathcal{A}_7^{(2)} = \frac{qG(r)}{2[1+NG(r)]}, \end{aligned} \tag{23}$$

All other fields are set to zero. In the above expressions, $G(r) = \frac{1}{4\omega_5 r^4}$, $N = M\cosh^2\frac{\delta}{2}$, $q = M\sinh\delta$. Here $\omega_5$ is the unit volume of the 5-sphere. The tension of the string is given by $T = N$. Using the asymptotic behavior of various fields, we find that the NS-NS two form charge ($Z$), the electric charge ($Q$) and the electric current ($J$) for the solution are given respectively by:

$$Z = N, \quad Q = q, \quad \text{and} \quad J = q. \tag{24}$$

Notice that the electric charge and current are same for the solution. One way to obtain this solution is to first embed eight dimensional heterotic string theory in type IIB string theory. Then we can translate the charged heterotic string solutions of [14] in terms of type IIB variables.

Now, using the symmetry of eight dimensional type IIB strings, one can construct an $SL(3,Z)$ multiplet of above string solution. We do not give this explicitly, since it is straightforward to write them down. We directly write down the charges $\mathbf{Z}'$ and and current $\mathbf{J}'$ that follow from the above configuration:

$$\begin{aligned} \mathbf{Z}' &= \begin{pmatrix} Z'^{(1)} \\ Z'^{(2)} \\ Z'^{(3)} \end{pmatrix} = N_{(z_1,z_2,z_3)} \begin{pmatrix} \cos\theta\cos\phi \\ \sin\theta\cos\phi \\ \sin\phi \end{pmatrix}, \\ \mathbf{J}' &= \begin{pmatrix} J'^{(1)} \\ J'^{(2)} \\ J'^{(3)} \end{pmatrix} = q_{(z_1,z_2,z_3)} \begin{pmatrix} -\sin\theta\cos\psi - \cos\theta\sin\phi\sin\psi \\ \cos\theta\cos\psi - \sin\theta\sin\phi\sin\psi \\ \cos\phi\sin\psi \end{pmatrix}, \end{aligned} \tag{25}$$

where $N_{(z_1,z_2,z_3)} = \sqrt{z_1^2+z_2^2+z_3^2}N$ and $q_{(z_1,z_2,z_3)} = \sqrt{z_1^2+z_2^2+z_3^2}q$. Parameters $\theta, \phi, \psi$ in equation (25) are the Euler angles, as mentioned earlier. We would now like

to identify $\mathbf{Z'} = (z_1, z_2, z_3)^T M \cosh^2\delta/2$. This, in turn, fixes part of the $SO(3)$ group parameters $\theta$ and $\phi$. Namely,

$$\cos\theta\cos\phi = \frac{z_1}{\sum_{i=1}^3 \sqrt{z_i^2}}, \;\; \sin\theta\cos\phi = \frac{z_2}{\sum_{i=1}^3 \sqrt{z_i^2}}, \;\; \sin\phi = \frac{z_3}{\sum_{i=1}^3 \sqrt{z_i^2}}. \tag{26}$$

Notice that in this way, $Z_{(1,0,0)}$ string corresponds to electrically charged F-string, $Z_{(0,1,0)}$ is an electrically charged D-string, and, $Z_{(0,0,1)}$ is a ten dimensional D-3 brane wrapped on two internal circles. The subscript on $Z$ in the last line denotes their $z$ quantum numbers. In a similar manner, we can define the electric charge of the configuration in (25) as $\mathbf{J'} = (q_1, q_2, q_3)^T M \sinh\delta$. However, $q$'s are not independent quantities. Rather, they are determined by $z$'s and one of the $SO(3)$ group parameter $\psi$. Explicitly, using (26) in (25) for currents, we get

$$\begin{pmatrix} J_1' \\ J_2' \\ J_3' \end{pmatrix} = \begin{pmatrix} \frac{z_2\sqrt{z_1^2+z_2^2+z_3^2}\cos\psi - z_1 z_3 \sin\psi}{\sqrt{z_1^2+z_2^2}} \\ \frac{z_1\sqrt{z_1^2+z_2^2+z_3^2}\cos\psi - z_2 z_3 \sin\psi}{\sqrt{z_1^2+z_2^2}} \\ \sqrt{z_1^2+z_2^2}\sin\psi \end{pmatrix}. \tag{27}$$

Here we note that one can get different $\mathbf{J'}$'s for different value of $\psi$.

In order to construct a junction configuration, one can consider a special class of solutions, namely where a $(z_1, 0, 0)$ and $(0, z_2, 0)$ strings meet. From the $\mathbf{Z}$ charge conservation, we see that resulting string must be a $(z_1, z_2, 0)$ string. Furthermore, in order to analyze the stability of a junction of three such strings we notice that their string tensions are given by expressions:

$$T_{(z_1,z_2,z_3)} = \sqrt{z_1^2 + z_2^2 + z_3^2}\, T_{(1,0,0)} \cosh^2\frac{\delta}{2}, \tag{28}$$

where $T_{(1,0,0)}$ is the tension of electrically neutral $(1,0,0)$ string, and, from (24), we see that it is given by $T_{(1,0,0)} = M$. Once again, since $\delta$ does not mix with $z_1$ and $z_2$, various angles between the strings in a network would be same as their electrically neutral counterparts. Beside $Z$ charge conservation and tension balance, our string junction have to satisfy other constraints as discussed before. One of them comes from electric current conservation. We notice that in general the electric charge is not conserved unless $\psi = 0$. Thus the only allowed charged string junction in this class is for $\psi = 0$, when

$$\mathbf{J'}_{(z_1,z_2,0)} = \mathbf{J'}_{(z_1,0,0)} + \mathbf{J'}_{(0,z_2,0)}. \tag{29}$$

This is the 'Kirchoff's law' for the junction. It simply says that the algebraic sum of the currents around the junction must be zero. At this stage, the restriction on $\psi$ for charge conservation might seem unnatural. However, we should notice that we started with a very special class of solutions (22). We thus believe that the restriction on $\psi$ is an artifact of restricting ourselves within this special class of configuration.

We expect that such restrictions can be avoided if we look for more general class of string junctions. Now, turning back to (23), we see that the seed solution that we started with has $\mathcal{A}_t^{(2)} = -\mathcal{A}_7^{(2)}$. This in turn, leads to a condition on the charge densities similar to the one in (29) for the currents.

Now, for charge string junctions satisfying the above conservation and stability criteria, we can put them together to construct a string network as in [5,15]. However, unlike in the previous cases, in our case these conditions only guarantee their classical stability properties. In addition, as in [5, 15], one has to examine supersymmetry property as well to find out if these are BPS string networks or not. In the later case, they will decay into other BPS states. It is of interest to examine if the corresponding final states are again string-networks and whether they are built out of charged or neutral strings. These statements can be made more precise by thinking of string networks on tori [5, 7]. Then the final mass formula for a 'particle-like' object will carry the overall factor $\cosh^2\frac{\delta}{2}$ appearing in the string tension in (28) which has a minimum at $\delta = 0$. One however needs a more careful study, including quantum corrections, to clarify this further.

We conclude by stating that one of our main motivation for studying charged, current carrying junction configurations and their networks is to set a framework for understanding entropy associated with the network when compactified on two-torus. Electrically charged networks that we discuss in this paper can be viewed as excitations over neutral networks. These excitations, in some examples, preserve a fraction of original supersymmetry. We believe (as was in the case of identifying string states associated with black hole entropy; see for example [19], [20]) that identification of the degrees of freedom for such excitations will play important role in understanding entropy associated with network on torus. We hope to return back to this issue in the future.

**Acknowledgements**: We have benefitted from discussions with Sunil Mukhi and Ashoke Sen.

# References


[1] J. Schwarz, *Lectures on Superstring and M theory dualities*, hep-th/9607201.

[2] O. Aharony, J. Sonnenschine and Y. Yankelowicz, *Nucl. Phys.* **B474** (1996) 309, hep-th/9603009; O. Aharony and A. Hanany, *Nucl. Phys.* **B504** (1997) 239, hep-th/9704170; O. Aharony, A. Hananny and B. Kol, *JHEP* **9801:002** (1998), hep-th/9710116.

[3] C. Callan and J. Maldacena, *Nucl. Phys.* **B513** (1998) 198, hep-th/9708147.

[4] K. Dasgupta and S. Mukhi, *Phys. Lett* **B423** (1998) 261, hep-th/9711094.

[5] A. Sen, *JHEP* **9803:005**, 1998, hep-th/9711130.

[6] M. Krogh and S. Lee, *Nucl. Phys.* **B516** (1998) 241, hep-th/9712050; Y. Matsuo and K. Okuyama, *Phys. Lett.* **B426** (1998) 294, hep-th/9712070.

[7] S. Bhattacharyya, A. Kumar and S. Mukhopadhyay, *Phys. Rev. Lett.* **81** (1998) 754, hep-th/9801141; A. Kumar, *JHEP* **03** (2000) 010, hep-th/0002150.

[8] S. Mukherji, *Mod. Phys. Lett.* **A13** (1998) 2819, hep-th/9805031.

[9] J.X. Lu and S. Roy, *Phys. Lett.* **B443** (1998) 167, hep-th/9807139.

[10] C. Fraser and T. Hollowood, *Phys. Lett.***B402** (1997) 106, hep-th/9704011.

[11] M. Gabardiel and B. Zwiebach, *Nucl. Phys.* **B518** (1998) 151, hep-th/9709013; M. Gabardiel, T. Hauer and B. Zwiebach, *Nucl. Phys.* **B525** (1998) 117, hep-th/9801205; O. DeWolfe and B. Zwiebach, *Nucl. Phys.* **B541** (1999) 509, hep-th/9804210.

[12] O. Bergman, *Nucl. Phys.* **525** (1998) 104, hep-th/9712211; O. Bergman and A. Fayyazuddin, *Nucl. Phys.* **B531** (1998) 108, hep-th/9802033; O. Bergman and B. Kol, *Nucl. Phys.* **B536** (1998) 149, hep-th/9804160.

[13] K. Hasimoto, H. Hata and N. Sasakura, *Phys. Lett.* **B431)** (1998) 303, hep-th/9803127; *Nucl. Phys.* **B535** (1998) 83, hep-th/9804164; T. Kawano and K. Okuyama, *Phys. Lett.* **B432** (1998) 338, hep-th/9804139; K. Li and S. Yi, *Phys. Rev* **D58** (1998) 066005, hep-th/9804174.

[14] A. Sen, *Nucl. Phys.* **B388** (1992) 457, hep-th/9206016.

[15] A. Kumar, *JHEP* **9912:001** (1999), hep-th/9911090.

[16] J. Schwarz, *Phys. Lett.* **B360** (1995) 13, hep-th/9508143.

[17] J.X. Lu and S. Roy, *Nucl. Phys* **B538** (1999) 149, hep-th/9805180.

[18] E. Cremmer, B. Julia, H. Lu and C.N. Pope, *Nucl. Phys.* **B258** (1998) 73, hep-th/9710119.

[19] C. Callan, A. Peet and J. Maldacena, *Nucl. Phys.* **B475** (1996) 645, hep-th/9510134.

[20] A. Dabholkar, J. Gountlett, J. Harvey and D. Waldram, *Nucl. Phys.* **B474** (1996) 85, hep-th/9511053.